

Quark and Lepton Mass Matrices from Horizontal U(1) Symmetry

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Abstract

In the simplest model of horizontal U(1) symmetry with one singlet added to the supersymmetric standard model, we systematically reconstruct quark mass matrices from the low-energy data to prove that there are only two mass matrices found by Binetruy et. al.. The same U(1) symmetry constrains the hierarchical structure of L-violating couplings, from which we build radiative neutrino mass matrices accommodating the solar and hot dark matter neutrino masses and mixing. We find a few patterns of acceptable charged lepton and neutrino mass matrices, most of which are consistent with large $\tan \beta \simeq m_t/m_b$ only.

1. Introduction

Understanding the hierarchies of fermion masses is one of the fundamental problems in particle physics. One way to explain the structure of fermion mass matrices is to assume a generation-dependent $U(1)$ symmetry, as originally explored in [1]. In this approach, the horizontal $U(1)$ symmetry constrains non-renormalizable couplings of quarks and leptons to certain $SU(3)_c \times SU(2)_L \times U(1)_Y$ singlet field. The hierarchical values of Yukawa couplings are given by some powers of the vacuum expectation value of the singlet spontaneously breaking the horizontal $U(1)$ symmetry.

Recent investigations [2]–[11] have been amounted to reveal some remarkable features of this class of models. First of all, a potential connection to string theory was pointed out in [3, 4, 5]. In the simplest model with only one singlet field, phenomenologically allowed quark mass hierarchies require the $U(1)$ to be anomalous. Then the cancellation of the anomalies may be the consequence of Green-Schwarz mechanism in string theories [12], a remarkable feature of which is the prediction of the mixing angle $\sin^2 \theta_W = 3/8$ [13]. It was also shown that the horizontal $U(1)$ symmetry may provide a solution to the Higgs mass problem (the μ -problem) [6, 7, 9]. More recently, detailed analyses have been made to prove that this class of models indeed can account for the structure of the CKM mixing matrix [8, 11]. In particular, for the simplest model with one singlet, only two sets of quark Yukawa matrices are found to be consistent with the experimental values [11]. In the first half of this paper, we will systematically reconstruct the quark Yukawa matrices starting from the experimental values to prove there are no other acceptable quark mass matrices.

Contrary to the quark sector, the lepton sector is not much constrained. Our aim in the second half is to single out acceptable mass matrices for the charged leptons and neutrinos. Even if there are not yet decisive evidences for non-zero neutrino masses, some strong indications from astronomical and cosmological observations have been widely discussed [14]. Theoretical efforts to understand neutrino mass structure in the context of horizontal abelian symmetry were recently made by several groups [15, 16, 11]. They mostly rely on the see-saw mechanism to generate light neutrino masses. In this paper, we will look for R-parity violating terms (in particular, L-violating terms) which are inherent in the minimal supersymmetric standard model (MSSM). Without an additional symmetry like R-parity, the MSSM allows the presence of L- or B-violating operators;

$$\Lambda L Q D, \quad \Lambda' L L E, \quad \Lambda'' U D D.$$

Proton stability imposes an extremely strong constraint on $\Lambda\Lambda''$. On the other hand, in the

presence of the above L-violating operators neutrino masses can be generated radiatively. In general, these masses are not very much constrained due to the large freedom in the coupling constants Λ and Λ' . As we will see, this situation changes significantly once a horizontal $U(1)$ symmetry determines the structure of the R-parity violating operator [17, 18]. Then, since the charge assignment is fixed to a large extent due to the known masses and anomaly cancellation the order of Λ , Λ' and the radiative neutrino mass matrices depends on a few charge values only. We will analyze these matrices and show that a few solutions compatible with the solar neutrino data and a hot dark matter neutrino mass exist. Simultaneously, the B-violating operators UDD can be forbidden by the $U(1)$ symmetry to ensure proton stability.

2. Reconstruction of quark mass matrices

We consider the supersymmetric standard model whose Yukawa structure originates from horizontal abelian symmetry $U(1)_X$. Taking the simplest form, we introduce only one singlet field ϕ whose $U(1)_X$ -invariant non-renormalizable couplings to quarks and leptons determine hierarchical patterns of the Yukawa matrices. We assume that non-renormalizable couplings are suppressed by the string scale (or Planck scale) M invoking a string origin of the model. The charges of $U(1)_X$ can be any integer. When ϕ carries charge N , spontaneous breaking of $U(1)_X$ due to the vacuum expectation value $\langle\phi\rangle$ leaves Z_N as an unbroken subgroup. For convenience we normalize the charge of ϕ to -1 and thus the charges of quarks, leptons and Higgses are integers divided by N . We assume that spontaneous breaking of $U(1)_X$ occurs slightly below the scale M giving rise to the Cabibbo angle $\epsilon \equiv \langle\phi\rangle/M \simeq 0.22$ in order to predict the experimental values of the CKM mixing angles [19].

The large mass of top quark leads us to assume that the (3,3)-component of the up-type quark Yukawa couplings is given by the renormalizable term $Q_3 U_3 H_2$ which implies $q_3 + u_3 + h_2 = 0$. We use the capital letters Q, U, D, L, E and $H_{1,2}$ to denote MSSM superfields, and the corresponding small letters denote their charges under $U(1)_X$. Depending on the value of $\tan\beta$, the bottom quark Yukawa term can be written as $Q_3 D_3 H_1 (\phi/M)^x$ where $x = q_3 + d_3 + h_1$ is a not too large positive integer. Note that $\tan\beta \simeq \epsilon^x m_t/m_b$. The excess charge matrices $y_{ij}^u \equiv q_{i3} + u_{j3}$ and $y_{ij}^d \equiv q_{i3} + d_{j3}$ (normalized by $y_{33}^{u,d} = 0$) determine the Yukawa matrices for the up- and down-type quarks

$$Y_{ij}^u \sim \epsilon^{y_{ij}^u} \theta(y_{ij}^u), \quad Y_{ij}^d \sim \epsilon^{x+y_{ij}^d} \theta(x+y_{ij}^d) \quad (1)$$

where $q_{ij} \equiv q_i - q_j$, etc. and the function θ is defined as $\theta(x) = 1$ when x is a *non-negative integer*, and $\theta(x) = 0$ otherwise. As noted in [8, 11], possible zeros in the Yukawa matrices

(1) are filled by non-minimal contributions to the kinetic term consistent with $U(1)_X$. Their major role is to change the naive mixing angles calculated from the Yukawa matrices (1). In the canonical basis corresponding to normalized kinetic terms, the corrected Yukawa matrices are [8]

$$\hat{Y}_{ij}^u \sim \sum_{l,m} \epsilon^{|q_{il}|+|u_{jm}|} Y_{lm}^u, \quad \hat{Y}_{ij}^d \sim \sum_{l,m} \epsilon^{|q_{il}|+|d_{jm}|} Y_{lm}^d, \quad (2)$$

where, e.g., $\epsilon^{|q_{il}|}$ vanishes when $|q_{il}|$ is non-interger.

Taking the dominant contribution, one obtains for the down-type; $\hat{Y}_{ij}^d = \epsilon^{x+n_{ij}^d}$ where

$$n_{ij}^d = \begin{cases} q_{i3} + d_{j3}, & \text{when } q_{i3} + d_{j3} + x \text{ is a non-negative integer} \\ \text{Min} [\hat{\theta}(|q_{il}|) + \hat{\theta}(|d_{jm}|) + \hat{\theta}(q_{l3} + d_{m3} + x)] - x, & \text{otherwise.} \end{cases} \quad (3)$$

Here Min on the second line picks up the smallest value varying l, m from 1 to 3. The function $\hat{\theta}$ assigns infinity when the argument is *negative or fractional* number and $\hat{\theta}(x) = x$ otherwise. As observed in [11], the expressions for n_{i3} and n_{3j} are simple:

$$n_{i3}^u = n_{i3}^d = |q_{i3}|, \quad n_{3j}^u = |u_{j3}|, \quad n_{3j}^d = |d_{j3}|. \quad (4)$$

We note that this property holds independently of the value of x assuming that the (3,3)-component is the largest: $n_{33} \leq n_{ij}$. From the property $n_{ij} \geq |y_{ij}|$ [11] and $y_{ij} = y_{i3} + y_{3j}$ one finds

$$\begin{aligned} n_{ij} &\leq n_{i3} + n_{3j} \\ n_{i3} &\leq n_{ij} + n_{3j} \\ n_{3j} &\leq n_{ij} + n_{i3} \end{aligned} \quad (5)$$

These inequalities $n_{ij}^{u,d}$ allow us to completely reconstruct the Yukawa matrices from the experimental values of the mass eigenvalues and the CKM matrix. In refs. [8, 11], two mass matrices are found to be acceptable. In the below we will give a systematic proof of their uniqueness.

The eigenvalues of the Yukawa matrices \hat{Y} determined by n_{ij} in (3) are ϵ^{ρ_1} , ϵ^{ρ_2} and 1 where

$$\begin{aligned} \rho_1 &= \text{Min} [n_{11} + n_{22}, n_{12} + n_{21}] - \rho_2 \\ \rho_2 &= \text{Min} [n_{11}, n_{22}, n_{12}, n_{21}] \end{aligned} \quad (6)$$

in case that $\rho_1 > \rho_2$ [4, 11]. The experimental values of $\rho_{1,2}$ extrapolated to the Planck scale are known to be $(\rho_1^u, \rho_2^u) = (8, 4)$ and $(\rho_1^d, \rho_2^d) = (4, 2)$ for the up- and down-type quarks,

respectively [20]. Notice that one can fix $n_{22} = \rho_2$ (namely, $n_{22}^u = 4$ and $n_{22}^d = 2$) by reordering q_{i3}, u_{i3} and d_{i3} and by correct hierarchical values between the first and the second generations. Then, one gets $n_{ij} \geq n_{22}$ for $i, j = 1, 2$. Another important property is that

$$y_{11} + y_{22} = y_{12} + y_{21} = \rho_1 + \rho_2 \quad (7)$$

since the determinant of the Yukawa matrix (1) should be regular and cannot change its value even after the kinetic correction. The regularity condition of Y_{ij} also shows that both of two elements in the same row or column of the left-upper 2×2 submatrix cannot be negative or fractional at the same time. From these properties and eq. (6), we arrive at the conclusion that e.g. the down-type excess charge matrices yielding $n_{22}^d = 2$ is characterized by one of the following two patterns:

$$\begin{aligned} y_{11}^d &= q_{13} + d_{13} = 4, & y_{22}^d &= q_{23} + d_{23} = 2; \\ y_{11}^d &= 8, & y_{22}^d &= -2 \quad \text{with} \quad q_{23} = 0, -1, -2. \end{aligned} \quad (8)$$

The latter condition comes from the fact that $n_{22}^d = |q_{23}| + |d_{23}| = 2$ when $y_{22}^d < 0$. Similar conclusion can be drawn also for the up-type quarks:

$$\begin{aligned} q_{13} + u_{13} &= 8, & q_{23} + u_{23} &= 4; \\ |q_{23}| + |u_{23}| &= 4 \quad \text{if} \quad q_{23} + u_{23} < 0. \end{aligned} \quad (9)$$

The above properties are useful to reconstruct quark mass matrices compatible with the CKM mixing angles.

The CKM matrix in the leading order can be parameterized in terms of three left-handed mixing angles as follows:

$$V_{CKM} = \begin{pmatrix} 1 & -S_{12}^- - S_{13}^u S_{23}^- & -S_{13}^- + S_{12}^u S_{23}^- \\ S_{12}^- + S_{13}^d S_{23}^- & 1 & -S_{23}^- - S_{12}^u S_{13}^- \\ S_{13}^- - S_{12}^d S_{23}^- & S_{23}^- + S_{12}^d S_{13}^- & 1 \end{pmatrix} \quad (10)$$

where $S_{ij}^- = S_{ij}^u - S_{ij}^d$ [21]. With the assumption $Y_{ij} \leq Y_{33}$, an order of magnitude calculation can be made to express the mixing angles S_{ij} in terms of the Yukawa elements [21]. In the model under consideration, the expressions become extremely simple. Iterative use of (5) reveals

$$\begin{aligned} s_{13} &= n_{13}, & s_{23} &= n_{23} \\ s_{12} &= \text{Min}[n_{12} - n_{22}, n_{11} + n_{21} - 2n_{22}] \end{aligned} \quad (11)$$

where $S_{ij} \sim \epsilon^{s_{ij}}$. From (4), we immediately get $s_{i3}^u = s_{i3}^d = |q_{i3}|$. Comparing the CKM matrix (10) to the experimental data [19], we find two possibilities:

$$\begin{aligned} s_{12}^u \text{ or } s_{12}^d = 1, & \quad s_{13}^u \text{ or } s_{13}^d = 3, & \quad s_{23}^u \text{ or } s_{23}^d = 2; \\ s_{12}^u \text{ and } s_{12}^d = 1, & \quad s_{13}^u \text{ and } s_{13}^d > 3, & \quad s_{23}^u \text{ or } s_{23}^d = 2. \end{aligned} \quad (12)$$

The above conditions can be rewritten as

$$\begin{aligned} |q_{23}| = 2, & \quad |q_{13}| = 3, & \quad s_{12}^u \text{ or } s_{12}^d = 1; \\ |q_{23}| = 2, & \quad |q_{13}| > 3, & \quad s_{12}^u = s_{12}^d = 1. \end{aligned} \quad (13)$$

It is now useful to realize that the condition $s_{12} = 1$ cannot be met by the term $s_{12} = n_{11} + n_{21} - 2n_{22}$ since $n_{11} \geq 2n_{22}$ (see (7)) and $n_{21} \geq n_{22} = \rho_2$. Therefore, one gets $n_{12} = s_{12} + \rho_2$ for $s_{12} = 1$. After some manipulation just with the down-type quark mass matrices satisfying (8), one finds that the second condition in (13) cannot be fulfilled. In order to examine the first case in (13), one tries out mass matrices satisfying $(q_{13}, q_{23}) = (\pm 3, \pm 2)$ under the conditions in (8) and (9). Then we find that s_{12}^u or $s_{12}^d = 1$ can only be obtained for $(q_{13}, q_{23}) = (3, 2)$ and $(-3, 2)$. These corresponds to the two mass matrices found in [11]. The first one has the excess charges $(q_{13}, q_{23}) = (3, 2)$, $(u_{13}, u_{23}) = (5, 2)$ and $(d_{13}, d_{23}) = (1, 0)$ producing the Yukawa matrix

$$(I) \quad \hat{Y}^u \sim \begin{pmatrix} \epsilon^8 & \epsilon^5 & \epsilon^3 \\ \epsilon^7 & \epsilon^4 & \epsilon^2 \\ \epsilon^5 & \epsilon^2 & 1 \end{pmatrix}, \quad \hat{Y}^d \sim \epsilon^x \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^1 & 1 & 1 \end{pmatrix}. \quad (14)$$

For the second one, $(q_{13}, q_{23}) = (-3, 2)$, $(u_{13}, u_{23}) = (11, 2)$ and $(d_{13}, d_{23}) = (7, 0)$; the resulting Yukawa matrix is

$$(II) \quad \hat{Y}^u \sim \begin{pmatrix} \epsilon^8 & \epsilon^5 & \epsilon^3 \\ \epsilon^{13} & \epsilon^4 & \epsilon^2 \\ \epsilon^{11} & \epsilon^2 & 1 \end{pmatrix}, \quad \hat{Y}^d \sim \epsilon^x \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^9 & \epsilon^2 & \epsilon^2 \\ \epsilon^7 & 1 & 1 \end{pmatrix}. \quad (15)$$

We also remark that the experimental values of the mass eigenvalues and the CKM mixing angles cannot be reproduced when any of y_{ij} is fractional.

3. Lepton mass matrices and R-parity violating couplings

The lepton sector is only weakly constrained compared to the quark sector. Furthermore, the MSSM allows L- and B-violating terms whose presence may destroy proton stability. The

usual wisdom is to assign R-parity to forbid both or either of them. As one can see, the horizontal $U(1)$ symmetry compatible with the experimental quark and lepton mass matrices still has large number of free charges. As a result, it is always possible to choose some fractional or negative excess charges so that undesirable R-parity violating operators can be discarded. One can also give large enough positive excess charges to suppress the proton decay. In the below we will investigate the case where L-violating operators are allowed but B-violating operators are sufficiently suppressed. The structure of the L-violating operators as well as the radiatively generated neutrino masses resulting from the $U(1)_X$ symmetry will be analyzed.

The measurement of solar neutrino deficit strongly indicates the neutrino mass and mixing;

$$m_{\nu_\alpha} \simeq 2 \times 10^{-3} \text{ eV}, \quad \theta_{e\alpha} \simeq 3 \times 10^{-2} \sim \epsilon^2 \quad (16)$$

which provides the right amount of resonance conversion $\nu_e \rightarrow \nu_\alpha$ ($\alpha = \mu, \tau$) inside the Sun [14]. Other hints for neutrino masses come from the deficit of atmospheric neutrinos and the need for hot dark matter in structure formation. Given hierarchical Yukawa matrices, it appears unnatural to accommodate all of three evidences which require nearly degenerate neutrino mass matrices [22]. Therefore we will analyze to what extent the $U(1)$ -constrained radiative neutrino mass matrices can provide the solar neutrino mass and mixing as well as a hot dark matter neutrino mass $m_\nu \sim 10 \text{ eV}$.

For the charged leptons, one gets the following Yukawa matrix elements:

$$Y_{ij}^e \sim \epsilon^{x+y_{ij}^e} \theta(x + y_{ij}^e) \quad \text{with} \quad y_{ij}^e = l_{i3} + e_{j3} \quad (17)$$

where we have used the b - τ unification at the Planck scale, $x = l_3 + e_3 + h_1 = q_3 + d_3 + h_1$. The couplings of the R-parity violating terms are

$$\begin{aligned} \Lambda_{ijk} &\sim \epsilon^{y_{ijk}} \theta(y_{ijk}) & \text{with} & \quad y_{ijk} = l_0 + l_{i3} + q_{j3} + d_{k3} \\ \Lambda'_{ijk} &\sim \epsilon^{y'_{ijk}} \theta(y'_{ijk}) & \text{with} & \quad y'_{ijk} = l_0 + l_{i3} + l_{j3} + e_{k3} \\ \Lambda''_{ijk} &\sim \epsilon^{y''_{ijk}} \theta(y''_{ijk}) & \text{with} & \quad y''_{ijk} = b_0 + u_{i3} + d_{j3} + d_{k3} \end{aligned}$$

where $l_0 \equiv l_3 + q_3 + d_3 = l_3 + l_3 + e_3$ and $b_0 \equiv u_3 + d_3 + d_3$. One has to bear in mind that $\Lambda'_{iij} = \Lambda''_{kll} \equiv 0$ due to the antisymmetric contractions. In the similar way as in the down quark sector, the experimental eigenvalues of the charged lepton mass matrix; $\rho_1 = 4, \rho_2 = 2$ [4] eliminate two free charges;

$$\begin{aligned} e_{13} &= 4 - l_{13}, \quad e_{23} = 2 - l_{23}; \\ \text{or} \quad e_{23} &= -2 - l_{23} \quad \text{with} \quad l_{23} = -2, -1, 0. \end{aligned} \quad (18)$$

From now on, we will concentrate on the first possibility as it turns out that the second one does not yield proper lepton mass matrices for our purpose. The number of free charges is further reduced by considering the anomaly-free conditions of $U(1)_X$. The anomalies corresponding to $SU(3)^2 - U(1)_X$, $SU(2)^2 - U(1)_X$, $U(1)_Y^2 - U(1)_X$ and $U(1)_Y - U(1)_X^2$ are

$$\begin{aligned}
A_3 &= \sum_i (2q_i + u_i + d_i) \\
A_2 &= \sum_i (3q_i + l_i) + (h_1 + h_2) \\
A_1 &= \sum_i \left(\frac{1}{3}q_i + \frac{8}{3}u_i + \frac{2}{3}d_i + l_i + 2e_i \right) + (h_1 + h_2) \\
A'_1 &= \sum_i (q_i^2 - 2u_i^2 + d_i^2 - l_i^2 + e_i^2) - (h_1^2 - h_2^2).
\end{aligned} \tag{19}$$

The desirable quark mass eigenvalues satisfying $\sum_i (q_{i3} + u_{i3}) = 12$, $\sum_i (q_{i3} + d_{i3}) = \sum_i (l_{i3} + e_{i3}) = 6$ is not consistent with $A_3 = A_2 = A_1 = 0$ [4]. But the MSSM with horizontal $U(1)_X$ symmetry may come from superstring theory which allows the Green-Schwarz anomaly cancellation mechanism [12]; $k_3 g_3^2 = k_2 g_2^2 = k_1 g_1^2 \propto g_{string}^2$. Here k_i are Kac-Moody levels of the gauge groups which are integers for non-abelian groups. For the anomaly cancellation, the ratio of the anomalies should follow $A_3 : A_2 : A_1 = k_3 : k_2 : k_1$. The gauge coupling unification in the MSSM still holds quite well even near the Planck scale, which implies $k_3 = k_2 = 3k_1/5 = 1$ [13]. Therefore, the Green-Schwarz mechanism indicates the relations among the anomalies: $A_3 : A_2 : A_1 = 1 : 1 : 5/3$.

Then the relation $\sum_i (q_{i3} + d_{i3}) = \sum_i (l_{i3} + e_{i3})$ results in $h_1 + h_2 = 0$ allowing the presence of the Higgs mass term $\mu H_1 H_2$ in the superpotential. In this case, we are facing with the μ -problem. In the context of horizontal abelian symmetry, one may have a natural solution by assuming a slight deviation of the hierarchical eigenvalues like $\rho_1^e = 5$ [6] or when R-symmetry is responsible for the hierarchy [9]. The conditions $A_3 = A_2 = 3A_1/5$ together with $A'_1 = 0$ leaves three charges free which are chosen to be l_0 , l_{13} and l_{23} . The other charges are given by

$$\begin{aligned}
u_3 &= -q_3 + h_1, \quad d_3 = x - q_3 - h_1, \quad e_3 = x - l_3 - h_1, \quad l_3 = l_0 - x + h_1, \\
q_3 &= 2 + \frac{2}{5}x - \frac{1}{5}(b_0 + 1) - \frac{1}{3}(q_{13} + q_{23}), \quad h_1 = \frac{4}{5}x - \frac{2}{5}(b_0 + 1) \\
b_0 &= \frac{1}{3}(l_{13} + l_{23} + 3l_0) - 1
\end{aligned} \tag{20}$$

considering the first case in (18). Now the couplings Λ , Λ' and Λ'' are functions of l_0 and l_{i3} .

The radiative neutrino masses (assuming minimal soft terms) due to the couplings Λ and

Λ' are

$$m_{ij}^\nu \simeq \frac{\mu \tan \beta + A}{8\pi^2 m_0^2} \text{Tr} [3(\Lambda_i) M_d^T (\Lambda_j) M_d^T + (\Lambda'_i) M_e^T (\Lambda'_j) M_e^T], \quad (21)$$

where $(\Lambda_i)_{jk} \equiv \Lambda_{ijk}$, etc. and A is the soft-SUSY breaking parameter of the trilinear terms and m_0 is the soft mass of squarks or sleptons. Taking the leading order terms inside the trace, one can pick out the integers p_{ij} and p'_{ij} as follows;

$$\begin{aligned} m_{ij}^\nu &\sim \frac{m_\tau^2}{8\pi^2 m_0} \left[3 \left(\frac{m_b}{m_\tau} \right)^2 \epsilon^{p_{ij}} + \epsilon^{p'_{ij}} \right] \\ \text{with } p_{ij} &= \text{Min} [y_{ilm} + y_{jkn} + y_{ln}^d + y_{km}^d] \\ \text{and } p'_{ij} &= \text{Min} [y'_{ilm} + y'_{jkn} + y_{ln}^e + y_{km}^e] \end{aligned} \quad (22)$$

where $\mu \simeq A \simeq m_0 \simeq 0.1 - 1$ TeV. It is understood to neglect negative or fractional y 's inside Min.

The expression (22) is valid at the Planck scale where $m_b/m_\tau \simeq 1$. Renormalizing M_d , M_e as well as Λ , Λ' down to the weak scale, one gets the significant effect; $\Lambda/\Lambda' \simeq 3$ like $m_b/m_\tau \simeq 3$. Therefore the neutrino mass matrix at the weak scale is given by

$$m_{ij}^\nu \sim \tilde{m} [\epsilon^{p_{ij}-3} + \epsilon^{p'_{ij}}] \quad (23)$$

where $\tilde{m} \simeq 0.4 - 0.04$ MeV.

Writing $m_{ij}^\nu \sim \tilde{m} \epsilon^{n_{ij}^\nu}$, it is simple to get the number n_{ij}^ν in the case where l_0 and $l_0 + l_{i3}$ are all positive integers. It is convenient to calculate the neutrino mass matrix in the original basis with non-canonical kinetic term. As l_0 , $l_0 + l_{i3}$ are positive integers, Λ_{i33} is present and gives the largest contribution to the neutrino mass matrix so that

$$n^\nu = 2l_0 + \begin{pmatrix} 2l_{13} & l_{13} + l_{23} & l_{13} \\ l_{13} + l_{23} & 2l_{23} & l_{23} \\ l_{13} & l_{23} & 0 \end{pmatrix} - 3. \quad (24)$$

When $\tilde{m} \simeq 0.04 - 0.4$ MeV, one can extract the following best values for l_{i3} , l_0 and b_0 ;

$$l_{13} = 5, \quad l_{23} = 3, \quad l_0 = 4(5), \quad b_0 = \frac{17}{3} \left(\frac{21}{3} \right) \quad (25)$$

which (for $\tilde{m} \simeq 0.4$ MeV) give $m_{\nu_\tau} \sim 20(10)$ eV, $m_{\nu_\mu} \sim 2(1) \times 10^{-3}$ eV and $\theta_{e\mu} \sim \epsilon^2$. Even though the mass of a hot dark matter neutrino is not precisely fixed, one hardly can allow one unit of deviation in l_0 since it leads to two order deviation (ϵ^2) in the neutrino masses

which is too far from the desired value. The same is also true for the solar neutrino mass. The resulting neutrino mass matrices are

$$m^\nu \sim \begin{pmatrix} \epsilon^{15} & \epsilon^{13} & \epsilon^{10} \\ \epsilon^{13} & \epsilon^{11} & \epsilon^8 \\ \epsilon^{10} & \epsilon^8 & \epsilon^5 \end{pmatrix} (\epsilon^2). \quad (26)$$

We have chosen tau-neutrino as the heaviest one. In either case in (25), the excess charge matrix and mass matrix in the canonical basis for charged leptons are given by

$$y^e = \begin{pmatrix} 4 & 4 & 5 \\ 2 & 2 & 3 \\ -1 & -1 & 0 \end{pmatrix}, \quad \hat{M}^e \sim \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^5 \\ \epsilon^2 & \epsilon^2 & \epsilon^3 \\ \epsilon^1 & \epsilon^1 & 1 \end{pmatrix}. \quad (27)$$

It is interesting to note that the charged lepton mass matrix in (27) allows *only* $x = 0$, that is, large $\tan \beta \simeq m_t/m_b$. This conclusion holds for either matrix (I) or (II) in (14) and (15). Furthermore, due to the fractional value of b_0 the B-violating operators UDD are forbidden. We also notice that higher dimensional operators like $QQQL$, $uude$ dangerous for proton stability are also forbidden by the same reason.

Now let us generalize the above calculation. Given any fractional numbers l_0, l_{i3} , one of the following cases may come out. First, some of them can be fractional leading to degenerate matrices with a 2×2 nontrivial submatrix only. Matrices of this type cannot lead to the correct solar neutrino and hot dark matter masses simultaneously and are therefore disregarded. Second, Λ and Λ' are allowed to generate proper neutrino masses. Then the operators UDD should be completely forbidden or highly suppressed to achieve proton stability; $\Lambda\Lambda'' < 10^{-26}$. A large suppression turns out to be inconsistent with the desired neutrino masses due to the last relation in (20). Therefore we have to choose b_0 fractional or smaller than $-\text{Max}[u_i + d_j + d_k]$ which forbids UDD completely. As a third case one might try to allow Λ' and Λ'' and forbid Λ (or make it sufficiently small) which would also ensure proton stability. As we will see, the wanted neutrino mass matrix does not permit this possibility.

To examine the above cases explicitly, we carry out a systematic calculation of neutrino mass matrices as follows. We start by calculating neutrino mass matrices in the original basis as in (22). Then we go to the canonical basis by performing a rotation K_l for the lepton doublets L_i ;

$$K_l = \begin{pmatrix} 1 & \epsilon^{|l_{12}|} & \epsilon^{|l_{13}|} \\ \epsilon^{|l_{12}|} & 1 & \epsilon^{|l_{23}|} \\ \epsilon^{|l_{13}|} & \epsilon^{|l_{23}|} & 1 \end{pmatrix}. \quad (28)$$

Next step is to rotate the lepton doublet fields to diagonalize the charged-lepton mass matrix. This can be done by the rotation matrix R_l ;

$$R_l = \begin{pmatrix} 1 & -s_{12}^e & 0 \\ s_{12}^e & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -s_{13}^e \\ 0 & 1 & 0 \\ s_{13}^e & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -s_{23}^e \\ 0 & s_{13}^e & 1 \end{pmatrix} \quad (29)$$

with $s_{i3}^e = n_{i3}^e$, $s_{12}^e = n_{12}^e - 2$

as can be seen from (11). Taking the leading order, the total rotation matrix $R = R_l K_l$ is given by $R_{ij} \sim \epsilon^{r_{ij}}$ where $r_{ii} = 0$ and

$$\begin{aligned} r_{12} &= r_{21} = \text{Min}[n_{12}^e - 2, |l_{12}|] \\ r_{13} &= \text{Min}[n_{12}^e - 2 + |l_{23}|, |l_{13}|] \\ r_{23} &= \text{Min}[n_{12}^e - 2 + |l_{13}|, |l_{23}|] \\ r_{31} &= |l_{13}|, \quad r_{32} = |l_{23}|. \end{aligned} \quad (30)$$

In most cases of interest Λ gives the dominant contribution for the radiative neutrino mass. For case (I) in (14), one finds no more acceptable neutrino mass matrices even for some negative values of l_0, l_{i3} . To see this, let us first note that the resultant neutrino mass matrix is only two-by-two if one of $l_0, l_0 + l_{i3}$ is smaller than $-\text{Max}[q_{i3} + d_{j3}]$ ($= -4$ in case (I)). Hence, $l_0, l_0 + l_{i3} \geq -\text{Max}[q_{i3} + d_{i3}]$ has to be met. As one can see the smallest value (ϵ^5) in the neutrino mass matrix results from $l_0, (l_0 + l_{i3}) = -4$. For this choice, the same radiative neutrino masses and mixing as in (26) can be obtained taking $l_{13} = 13, l_{23} = 11$ and $l_0 = -4$. However, this leads to $b_0 = 3$ and therefore to a too large Λ'' . On the other hand, in case (II), we find one more neutrino mass matrix, which has negative integers l_0 and $l_0 + l_{23}$. When the tau neutrino is the heaviest, the explicit pattern is given by

$$l_{13} = 19, \quad l_{23} = -3, \quad l_0 = -4(-3), \quad b_0 = \frac{1}{3}(\frac{4}{3}) \quad (31)$$

for $\tilde{m} = 0.04 - 0.4$ MeV. The resultant neutrino mass matrix is

$$m^\nu \sim \begin{pmatrix} \epsilon^{27} & \epsilon^{13} & \epsilon^{16} \\ \epsilon^{13} & \epsilon^{11} & \epsilon^8 \\ \epsilon^{16} & \epsilon^8 & \epsilon^5 \end{pmatrix} (\epsilon^2). \quad (32)$$

As the corresponding charged lepton mass matrix in the canonical basis we obtain

$$y^e = \begin{pmatrix} 4 & 24 & 19 \\ -18 & 2 & -3 \\ -15 & 5 & 0 \end{pmatrix}, \quad \hat{M}^e \sim \epsilon^x \begin{pmatrix} \epsilon^4 & \epsilon^{24} & \epsilon^{19} \\ \epsilon^{18} & \epsilon^2 & \epsilon^3 \\ \epsilon^{15} & \epsilon^5 & 1 \end{pmatrix}. \quad (33)$$

Notice that this charge matrix can be consistent with $x = 0, 1, 2$ allowing also small $\tan \beta$.

The same calculation can be done with the muon neutrino as the heaviest. For both cases (I) and (II), the values $(l_{13}, l_{23}) = (2, -3)$, $l_0 = 7$ (8) and $b_0 = \frac{17}{3}$ ($\frac{20}{3}$) give rise to the following neutrino mass matrix in the M^e -diagonal canonical basis,

$$m^\nu \sim \begin{pmatrix} \epsilon^{15} & \epsilon^{10} & \epsilon^{13} \\ \epsilon^{10} & \epsilon^5 & \epsilon^8 \\ \epsilon^{13} & \epsilon^8 & \epsilon^{11} \end{pmatrix} (\epsilon^2). \quad (34)$$

It differs from (26) just by an exchange of the second and the third generation. In the charged lepton sector this solution is characterized by

$$y^e = \begin{pmatrix} 4 & 7 & 2 \\ -1 & 2 & -3 \\ 2 & 5 & 0 \end{pmatrix}, \quad \hat{M}^e \sim \epsilon^x \begin{pmatrix} \epsilon^4 & \epsilon^7 & \epsilon^2 \\ \epsilon^5 & \epsilon^2 & \epsilon^3 \\ \epsilon^2 & \epsilon^5 & 1 \end{pmatrix}. \quad (35)$$

For case (II), we find one more solution with $(l_{13}, l_{23}) = (22, 3)$, $l_0 = -7$ (-6) and $b_0 = \frac{1}{3}$ ($\frac{4}{3}$) yielding the neutrino mass matrix

$$m^\nu \sim \begin{pmatrix} \epsilon^{27} & \epsilon^{16} & \epsilon^{13} \\ \epsilon^{16} & \epsilon^5 & \epsilon^8 \\ \epsilon^{13} & \epsilon^8 & \epsilon^{11} \end{pmatrix} (\epsilon^2), \quad (36)$$

and the excess charge and mass matrix of charged leptons

$$y^e = \begin{pmatrix} 4 & 21 & 22 \\ -15 & 2 & 3 \\ -18 & -1 & 0 \end{pmatrix}, \quad \hat{M}^e \sim \epsilon^x \begin{pmatrix} \epsilon^4 & \epsilon^7 & \epsilon^{12} \\ \epsilon^{19} & \epsilon^2 & \epsilon^3 \\ \epsilon^{18} & \epsilon^1 & 1 \end{pmatrix}. \quad (37)$$

When the muon neutrino is the heaviest, only $x = 0$ is allowed as can be seen from (35) and (37).

4. Conclusion

We examined the model of horizontal $U(1)$ symmetry with one singlet added to MSSM, which explains the desired hierarchies of quarks and leptons. This model turns out to be quite constrained by the experimental values of quark masses and mixing. We systematically reconstructed the quark mass matrices to prove that there are only two acceptable patterns of quark Yukawa structures.

We extended the analysis to the charged leptons and neutrinos. Combined use of the experimental values of the charged lepton masses and Green-Schwarz mechanism of the anomaly cancellation of $U(1)$ reduces many free charges to three (l_{13}, l_{23}, l_0) in the lepton sector. Depending on their values, the presence of L- or B-violating operators can be controlled. Concentrating on L-violating operators (while suppressing B-violating operators for proton stability) we singled out a few patterns of the charged lepton and neutrino mass matrices which accommodate the solar and hot dark matter neutrino masses and mixing. Interestingly, only large $\tan\beta = m_t/m_b$ ($x = 0$) is allowed when the muon neutrino is the heaviest. When the tau-neutrino is the heaviest, the quark mass matrix (I) allows large $\tan\beta$ and only the pattern (II) is consistent with both large and small $\tan\beta$ ($x = 0, 1, 2$).

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